SSX scalings

In the spirit of my last note on the dimensionless Ohms law, here's a set of scalings for various parameters of interest for SSX. The idea is to have a set of formulae for quick calculations. These are all worth checking on your own. Let's set some nominal values for SSX:

$$\ell_{0.1m} \equiv 0.1\ m$$

$$B_{0.1T} \equiv 0.1\ T$$

$$T_{10} \equiv 10\ eV$$

$$n_{14} \equiv 10^{14}\ cm^{-3}$$

Using these, we can write down a few simple formulae:

$$\rho_i = 0.32\ cm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\rho_e = 0.075\ mm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\lambda_D = 2.35\ \mu m \frac{T_{10}^{1/2}}{n_{14}^{1/2}}$$

$$\beta \equiv \frac{W_{kin}}{W_{mag}} = 0.03 \frac{n_{14} T_{10}}{B_{0.1T}^2}$$

$$N_{particles} \cong 10^{19} n_{14}$$

$$W_{mag} \cong 0.4\ kJ\ B_{0.1T}^2$$

$$f_{ci} = 1.52\ MHz\ B_{0.1T}$$

$$f_{ce} = 2.8\ GHz\ B_{0.1T}$$

$$f_{pe} = 90\ GHz\ n_{14}^{1/2}$$

What this means is that typically $\rho_i \cong 0.32\ cm$ in SSX. If T_i is $4\times$ larger, ie. $T_i = 40\ eV$, then ρ_i is $2\times$ larger, ie $0.64\ cm$. Note that ρ_i also scales like $\sqrt{M_{ion}}$ so the carbon ions we're measuring have a gyroradius $\sqrt{12} = 3.5\times$ larger than protons at the same temperature. A typical DeBye length in SSX is a few microns and higher densities than 10^{14} only make it smaller.

$$v_A = 22 \frac{cm}{\mu s} \frac{B_{0.1T}}{n_{14}^{1/2}}$$

$$\text{where } 22\ \frac{cm}{\mu s} = 220\ \frac{km}{s}$$

$$\tau_A = \frac{\ell}{v_A} = 0.46\ \mu s \frac{\ell_{0.1m} n_{14}^{1/2}}{B_{0.1T}}$$

$$v_i = \sqrt{\beta} v_A$$

$$\delta_i = \frac{c}{\omega_{pi}} = \frac{2.3\ cm}{\sqrt{n_{14}}}$$

$$\delta_e = \frac{c}{\omega_{pe}} = \frac{0.53\ mm}{\sqrt{n_{14}}}$$

Now, if we invoke classical Spitzer resistivity, which is based on Coulomb collisions between ions and electrons, we have:

$$\eta = 5.15 \times 10^{-5} \frac{Z\ ln\lambda}{T_e^{3/2}} \Omega\ m$$

where $ln\lambda$ is about 10 for SSX, Z is about unity, and T_e is expressed in eV. Plugging in the values for SSX at 10 eV we get a scaled resistivity:

$$\eta_{10}^{SSX} = 1.6 \times 10^{-5}\ T_{10}^{-3/2}\ \Omega\ m$$

Using this, we can calculate all the SSX parameters that depend on resistivity.

$$\tau_{res} \equiv \frac{\mu_0 \ell^2}{\eta} = 770\ \mu s\ \ell_{0.1m}^2 T_{10}^{3/2}$$

$$S \equiv \frac{\tau_{res}}{\tau_A} = 1674\ \frac{\ell_{0.1m} T_{10}^{3/2} B_{0.1T}}{n_{14}^{1/2}}$$

$$R_m = 76\ v_{cm/\mu s}\ \ell_{0.1m} T_{10}^{3/2}$$

$$\tau_{whistler} = \frac{\ell^2}{\delta_e^2 \omega_{ce}} = 2\ \mu s\ \frac{n_{14}}{B_{0.1T}}$$

Stuff you can do if you have these formulae memorized include quickly comparing our parameters to others. Our $\delta_i = 2.3\ cm$. The magnetosphere plasma density is 10 cm^{-3} or 10^{13} smaller than ours. Since $\sqrt{10^{13}} \cong 3 \times 10^6$, the magnetosphere ion inertial scale (important for reconnection there) must be 3×10^6 bigger than ours or about 10^7 cm or 100 km (which is right). Spacecraft measurements (Polar and Cluster, see Mozer PRL) show a reconnection layer in the magnetosphere about that scale.