Interesting relationships between plasma lengths

Important length scales in plasma physics can often be represented as a ratio of a velocity to a frequency. For the ions with thermal speed v_i we have for the Larmor radius and ion inertial length:

$$\rho_i = \frac{v_i}{\omega_{ci}}$$

$$\delta_i = \frac{v_A}{\omega_{ci}} = \frac{c}{\omega_{pi}}$$

From this we immediately find:

$$\frac{\rho_i}{\delta_i} = \frac{v_i}{v_A} = \sqrt{\beta}$$

From the ion inertial length equations we get:

$$\frac{v_A}{c} = \frac{\omega_{ci}}{\omega_{pi}}$$

The ratio of cyclotron to plasma frequency for the ions isn't so interesting. More useful is that ratio for the electrons:

$$\frac{\omega_{ce}}{\omega_{pe}} = \sqrt{\frac{M_i}{m_e}}\frac{\omega_{ci}}{\omega_{pi}}$$

For SSX at 10^{15} and 0.1 T we have:

$$\frac{\omega_{ce}}{\omega_{pe}} = \frac{1}{100} = \frac{v_A}{c}\sqrt{\frac{M_i}{m_e}} = \frac{70\ km/s}{300,000\ km/s} \times 43$$

Since $\omega_{pe} \gg \omega_{ce}$, SSX is referred to as "over-dense".

For the electrons with thermal speed v_e we have for the Larmor radius, electron inertial length, and Debye length:

$$\rho_e = \frac{v_e}{\omega_{ce}}$$

$$\delta_e = \frac{v_{Ae}}{\omega_{ce}} = \frac{c}{\omega_{pe}}$$

$$\lambda_D = \frac{v_e}{\omega_{pe}}$$

The electron Alfven speed isn't so interesting but from the inertial and Debye length formulas we get:

$$\left(\frac{\lambda_D}{\delta_e}\right)^2 = \frac{kT_e}{m_e c^2}$$