SSX interferometer notes
MB, November 2021

Preliminaries: The fundamental idea of plasma interferometry is that electromagnetic radiation (light) that passes through plasma has a wavelength different than it has in air or vacuum. The wavelength is actually longer in plasma. The index of refraction in plasma can be written:

$$N_{plasma} = 1 - \frac{\omega_p^2}{2\omega_{HeNe}^2} = 1 - \frac{n}{2n_c}$$

where $\omega_p^2 = n_e e^2/\epsilon_0 m_e$ is the plasma frequency, and ω_{HeNe} is the frequency of the radiation. We use a HeNe laser, so the wavelength is $\lambda = c/f = 632.8 \times 10^{-9}\ m$. It is sometimes helpful to write the laser frequency in terms of a "critical density" (n_c) ie a density such that the plasma frequency at n_c equals the laser frequency $(n_c = \epsilon_0 m_e \omega_{HeNe}^2/e^2)$. This formula works as long as the laser frequency is large compared to the plasma frequency. For us, the ratio $\omega_{HeNe}/\omega_p \geq 100$, so we're good. Note also that $\omega_{pe}/\omega_{ce} \geq 100$ also so there is a large separation between electron magnetic fluctuations, electron electric fluctuations, and laser light fluctuations.

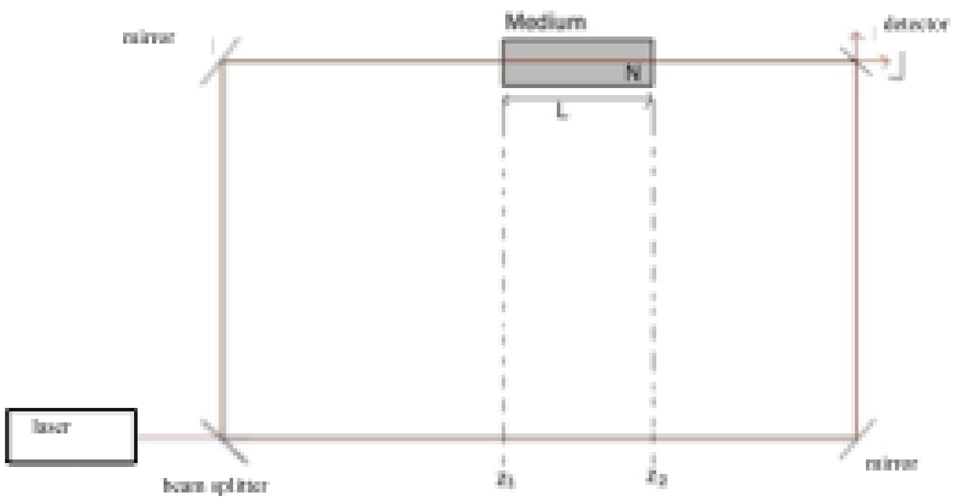

Figure 1: Typical Mach-Zehnder interferometer set up. The scene and reference paths are identical except for the path L through the plasma. For SSX, $L = 0.016\ m$.

Details: Since $\lambda_{HeNe} = 0.6\ \mu m$ and our path length is comparable to $0.6\ m$, we can think of there being about 10^6 wavelengths across the path.

With SSX plasma, and this interferometer, we change the path length by about 1 wavelength (ie one fewer wavelength in the plasma than in the vacuum). The path difference can be written:

$$\Delta s = \int_{z1}^{z2} N_{air} dz - \int_{z1}^{z2} N_{plasma} dz$$

Using the first formula and noting that $N_{air} \approx 1$, and noting that the ratio of ϕ compared to 2π is the same as the ratio of Δs to the wavelength λ, we get:

$$\phi = \frac{\pi}{\lambda n_c} \int_{z1}^{z2} n(z) dz = \frac{\lambda e^2}{4\pi c^2 \epsilon_0 m_e} \int_{z1}^{z2} n(z) dz$$

If you plug in the HeNe wavelength, electron charge and mass, ϵ_0, and speed of light (all MKS of course), you get for our typical peak density of $n = 10^{22}\ m^{-3}$ and a path length of $L = 0.16\ m$:

$$\phi_{22} = 2.85 \text{ radians}$$

Since a phase shift of $\pi/2 = 1.57$ corresponds to one excursion from dark to bright (ie the full envelope), 2.85 radians corresponds to about twice the envelope voltage (ie about 1 volt for us, if the alignment is good). We typically measure $n = 10^{21}\ m^{-3}$, so less than one fringe excursion.

References:

Hutchinson, Principles of Plasma Diagnostics