Coldwave summary (mb): These calculations are appropriate for Goldston 18.1, 2, 9 and Bellan 5.6. It turns out that $\omega_c/\omega_p = 312B/\sqrt{n}$ (in the usual units) so you can work out different regimes yourself. Notice that $\omega_c/\omega_p = 1$ corresponds equally well to $B = 1 \text{ G}, 10^5$ (ionosphere, say) as to $1 \text{ T}, 10^{13}$ (NSTX, say). We’ll work in the units where $\omega_p = 1$.

$\omega_c = 2\omega_p$ (big tokamak $2 \text{ T}, 10^{13}$):

$\omega_c = 2$ (resonance at $\theta = 0$)... $\Omega_c = 2/1837 = 1.09 \times 10^{-3}$ (resonance at $\theta = 0$)... $\Omega_p = 1/43 = 0.023$.

$\omega_L = (1/2)[-2 + \sqrt{4 + 4}] = -1 + \sqrt{2} = 0.414$... cutoff (Goldston, 17.19)

$\omega_R = (1/2)[2 + \sqrt{4 + 4}] = 1 + \sqrt{2} = 2.414$... cutoff

$\omega_p = 1$... cutoff

$\frac{3}{\Omega_L} = 1837 + \left(\frac{2 \times 2}{1837}\right)^{-1} \rightarrow \omega_{LH} = 0.0208...$ resonance at $\theta = 90^\circ$. (Goldston, 18.48)

$\omega_{UH} = 1 + 4 \rightarrow \omega_{UH} = \sqrt{5} = 2.236...$ resonance at $\theta = 90^\circ$. (Goldston, 17.9)

$$\frac{c}{v_{Alf}} = \frac{\Omega_p}{\Omega_c} = \frac{1}{43} \times \frac{1837}{2} = 21.5$$

... L, R waves at $\theta = 0$ (look at $(\omega, k) = (0.0001, 0.001)$).

$\omega_c = 0.5\omega_p$ (small tokamak $5 \text{ kG}, 10^{13}$):

$\omega_c = 0.5$ (resonance at $\theta = 0$)... $\Omega_c = 1/(2 \times 1837) = 2.72 \times 10^{-4}$ (resonance at $\theta = 0$)... $\Omega_p = 1/43 = 0.023$.

$\omega_L = (1/2)[-0.5 + \sqrt{1/4 + 4}] = 0.78...$ cutoff

$\omega_R = (1/2)[0.5 + \sqrt{1/4 + 4}] = 1.28...$ cutoff

$\omega_p = 1...$ cutoff

$\frac{1}{\Omega_L} = 1837 + \left(\frac{1}{2 \times 2 \times 1837}\right)^{-1} \rightarrow \omega_{LH} = 0.0104...$ resonance at $\theta = 90^\circ$.

$\omega_{UH} = 1 + 1/4 \rightarrow \omega_{UH} = \sqrt{5}/4 = 1.12...$ resonance at $\theta = 90^\circ$.

$$\frac{c}{v_{Alf}} = \frac{\Omega_p}{\Omega_c} = \frac{1}{43} \times 2 \times 1837 = 2 \times 43 = 86$$

... L, R waves at $\theta = 0$ (look at $(\omega, k) = (0.00001, 0.001)$).

$\omega_c = 10\omega_p$ (Q-machine $1 \text{ kG}, 10^9$): $(\omega, k) = (20, 20)$

$\omega_c = 10$ (resonance at $\theta = 0$)... $\Omega_c = 10/1837 = 5.44 \times 10^{-3}$ (resonance at $\theta = 0$)... $\Omega_p = 1/43 = 0.023$. 

\[ \omega_L = \frac{1}{2} \left[ -10 + \sqrt{100 + 4} \right] = 0.099 \ldots \text{ cutoff} \]
\[ \omega_R = \frac{1}{2} \left[ 10 + \sqrt{100 + 4} \right] = 10.1 \ldots \text{ cutoff} \]
\[ \omega_p = 1 \ldots \text{ cutoff} \]
\[ \frac{1}{\omega_{LH}^2} = 1837 + \left( \frac{10 \times 10}{1837} \right)^{-1} \rightarrow \omega_{LH} = 0.0232 \] (0.0238 if we use the more exact formula) ... resonance at \( \theta = 90^\circ \).
\[ \omega_{UH}^2 = 1 + 100 \rightarrow \omega_{UH} = \sqrt{101} = 10.05 \ldots \text{ resonance at } \theta = 90^\circ. \]
\[ \frac{c}{v_{Alf}} = \frac{\Omega_p}{\Omega_c} = \frac{1}{43} \times 1837 = 4.3 \]

... L, R waves at \( \theta = 0 \) (look at \((\omega, k) = (0.0001, 0.001)) \).

\[ \omega_c = 0.1 \omega_p \] ("SSX" 1 kG, 10\(^{13}\):)
\[ \omega_c = 0.1 \] (resonance at \( \theta = 0 \)) ... \( \Omega_c = 1/(10 \times 1837) = 5.44 \times 10^{-5} \)
(resonance at \( \theta = 0 \)) ... \( \Omega_p = 1/43 = 0.023 \).
\[ \omega_L = \frac{1}{2} \left[ -0.1 + \sqrt{0.01 + 4} \right] = 0.95 \ldots \text{ cutoff} \]
\[ \omega_R = \frac{1}{2} \left[ 0.1 + \sqrt{0.01 + 4} \right] = 1.05 \ldots \text{ cutoff} \]
\[ \omega_p = 1 \ldots \text{ cutoff} \]
\[ \frac{1}{\omega_{LH}^2} = 1837 + \left( \frac{1}{10 \times 1837} \right)^{-1} \rightarrow \omega_{LH} = 2.32 \times 10^{-3} \ldots \text{ resonance at } \theta = 90^\circ. \]
\[ \omega_{UH}^2 = 1 + 0.01 \rightarrow \omega_{UH} = \sqrt{1.01} = 1.005 \ldots \text{ resonance at } \theta = 90^\circ. \]
\[ \frac{c}{v_{Alf}} = \frac{\Omega_p}{\Omega_c} = \frac{1}{43} \times (10 \times 1837) = 430 \]

... L, R waves at \( \theta = 0 \) (look at \((\omega, k) = (0.00001, 0.001)) \).

Recall again that:

\[ \frac{v_A^2}{c^2} = \frac{\Omega_p^2}{\Omega_c^2} = \frac{B^2}{c^2 \mu_0 M n} = \frac{B^2 \epsilon_0}{M n} \]

so

\[ \epsilon_\perp = \epsilon_0 \left( 1 + \frac{M n}{B^2 \epsilon_0} \right) \]

which is the low-frequency "perpendicular dielectric constant" (eq 4.19 and 18.29).

Recall also...

\[ \omega_{R,L} = \left[ \pm \omega_c + \sqrt{\omega_c^2 + 4 \omega^2} \right]/2 \] (17.19)
\[ \omega_{LH}^2 \cong \Omega_p^{-2} + (\Omega_c \omega_c)^{-1} \] (18.48)
\[ \omega_{UH}^2 = \omega_p^2 + \omega_c^2 \] (17.9)