SSX scalings
MHD wind tunnel 2019

Here's a set of scalings for various parameters of interest for SSX in the compact wind tunnel configuration (volume is 20 liters). The idea is to have a set of formulae for quick calculations. These are all worth checking on your own. Let's set some nominal values for SSX:

$$\ell_{0.1m} \equiv 0.1\ m$$

$$B_{0.1T} \equiv 0.1\ T$$

$$T_{10} \equiv 10\ eV$$

$$n_{15} \equiv 10^{15}\ cm^{-3} = 10^{21}\ m^{-3}$$

Using these, we can write down a few simple formulae:

$$\rho_i = 3.2\ mm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\rho_e = 0.075\ mm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\lambda_D = 0.74\ \mu m \frac{T_{10}^{1/2}}{n_{15}^{1/2}}$$

$$\beta \equiv \frac{W_{kin}}{W_{mag}} = 0.4 \frac{n_{15} T_{10}}{B_{0.1T}^2}$$

$$N_{particles} \cong 2 \times 10^{19} n_{15}$$

$$W_{mag} \cong 0.08\ kJ\ B_{0.1T}^2$$

$$f_{ci} = 1.52\ MHz\ B_{0.1T}$$

$$f_{ce} = 2.8\ GHz\ B_{0.1T}$$

$$f_{pe} = 280\ GHz\ n_{15}^{1/2}$$

What this means is that typically $\rho_i \cong 3.2\ mm$ in SSX. If T_i is $4\times$ larger, ie. $T_i = 40\ eV$, then ρ_i is $2\times$ larger, ie $6.4\ mm$. Note that ρ_i also scales like $\sqrt{M_{ion}}$ so the carbon ions we're measuring have a gyroradius $\sqrt{12} = 3.5\times$ larger than protons at the same temperature. A typical DeBye length in SSX is a micron and higher densities than 10^{15} only make it smaller.

$$v_A = 7\ \frac{cm}{\mu s}\frac{B_{0.1T}}{n_{15}^{1/2}}$$

$$\text{where } 7\ \frac{cm}{\mu s} = 70\ \frac{km}{s}$$

$$\tau_A = \frac{\ell}{v_A} = 1.4\ \mu s \frac{\ell_{0.1m} n_{15}^{1/2}}{B_{0.1T}}$$

$$v_i = \sqrt{\beta} v_A$$

$$v_{sound} = 40\ km/s\ T_{10}^{1/2}$$

$$\delta_i = \frac{c}{\omega_{pi}} = \frac{7.2\ mm}{\sqrt{n_{15}}}$$

$$\delta_e = \frac{c}{\omega_{pe}} = \frac{0.17\ mm}{\sqrt{n_{15}}}$$

Note that the ratio of skin depths $\delta_i/\delta_e = \sqrt{M_p/m_e} = 43$. Now, if we invoke classical Spitzer resistivity, which is based on Coulomb collisions between ions and electrons, we have:

$$\eta = 5.15 \times 10^{-5} \frac{Z\ ln\lambda}{T_e^{3/2}} \Omega\ m$$

where $ln\lambda$ is about 10 for SSX, Z is about unity, and T_e is expressed in eV. Plugging in the values for SSX at 10 eV we get a scaled resistivity:

$$\eta_{10}^{SSX} = 1.6 \times 10^{-5}\ T_{10}^{-3/2}\ \Omega\ m$$

Using this, we can calculate all the SSX parameters that depend on resistivity.

$$\tau_{res} \equiv \frac{\mu_0 \ell^2}{\eta} = 770\ \mu s\ \ell_{0.1m}^2 T_{10}^{3/2}$$

$$S \equiv \frac{\tau_{res}}{\tau_A} = 530\ \frac{\ell_{0.1m} T_{10}^{3/2} B_{0.1T}}{n_{15}^{1/2}}$$

$$R_m = 76\ v_{cm/\mu s}\ \ell_{0.1m} T_{10}^{3/2}$$

Stuff you can do if you have these formulae memorized include quickly comparing our parameters to others. Our $\delta_i = 0.72\ cm$. The magnetosphere plasma density is 10 cm^{-3} or 10^{14} smaller than ours. Since $\sqrt{10^{14}} = 10^7$, the magnetosphere ion inertial scale (important for reconnection there) must be 10^7 bigger than ours or about 10^7 cm or 100 km (which is right). Spacecraft measurements (Polar and Cluster, see Mozer PRL) show a reconnection layer in the magnetosphere about that scale.