

## Diffusivity (connection between $\delta_e$ and $\eta$ )

**Collisions:** The drag force on an electron due to collisions with ions comes from:

$$\mathbf{F}_{drag} = \nu_{ei} m_e \mathbf{u}_{rel} = e \mathbf{E}$$

This is just saying that the electric force is being balanced by collisions.

The collision frequency has the form:

$$\nu_{ei} = \frac{n_i Z^2 e^4 \ln \lambda}{4\pi \epsilon_0^2 m^2 v^3}$$

where the  $v^{-3}$  dependence is evident (see earlier notes).

Now we remember Ohm's law:  $\mathbf{E} = \eta \mathbf{J}$  and  $\mathbf{J} = ne \mathbf{u}_{rel}$  so (combining the expressions for E):

$$\mathbf{E} = \eta ne \mathbf{u}_{rel} = \nu_{ei} m_e \mathbf{u}_{rel} / e$$

so we have an expression for the Spitzer resistivity:

$$\eta = \frac{m_e \nu_{ei}}{ne^2}$$

**Electron inertia:** The electron plasma frequency (in MKS) is (see earlier notes):

$$\omega_{pe}^2 = \frac{ne^2}{\epsilon_0 m_e}$$

The so-called electron inertial length is defined:  $\delta_e \equiv \frac{c}{\omega_{pe}}$  We can construct a diffusivity (with dimensions *length*<sup>2</sup>/*time*) using the collision frequency:  $\delta_e^2 \nu_{ei}$ . We can construct a fluid resistive diffusivity from the Spitzer formula above:  $\eta/\mu_0$ . Comparing these two, we find that they are the same thing!

$$\eta/\mu_0 = \frac{m_e \nu_{ei}}{ne^2 \mu_0} = \delta_e^2 \nu_{ei} = \frac{c^2 \epsilon_0 m_e \nu_{ei}}{ne^2}$$

This is interesting. It says that the MHD fluid resistivity can be thought of as electron diffusion at the electron inertial scale.