Simple model to explain electron heating
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Introduction:

The goal is to explain figure 8 of Vernon's paper with a simple model. The ingredients of the model should include (1) T_e increases from a baseline of 20 eV to 35 eV in 40 μs. There's a 10 μs delay before a linear ramp to 35 eV in 30 μs. A linear ramp suggests a constant power source (rather than an impulse or constant temperature bath). It might be hard to explain the delay with a simple model. (2) We know there's some activity at the midplane reconnection zone as early as $t = 30\ \mu s$ from IDS and SXR (figures 9 and 12) so our mechanism should turn on at $t = 30\ \mu s$ (even if the effect on T_e isn't immediate). (3) Since the excitation time for our carbon ions is so fast (0.2 μs), it is likely that not all the electrons in the layer are energized (otherwise we'd see a bigger effect immediately). We'll assume that most of the electrons in the layer are cold bulk electrons (that are slowly heated) with a small component energized by reconnection.

Something that won't work is simple diffusion. We consider a 1D diffusion problem (insulated cylinder with insulated ends) with a heat source in the middle. A narrow gaussian-shaped layer ($\sigma = 0.022\ m$ and FWHM $= 0.05\ m$) with peak temperature of 100 eV simply decays monotonically to 10 eV. One could imagine a layer that expands rapidly and adiabatically but that doesn't work either (not enough thermal energy in the layer to heat the whole plasma). In the end, we need a heat source that's turned on and stays on for 10's of microseconds.

Simple Model:

We need to fix a few parameters for our model. Most of these can be determined from my SSX scaling sheet on the SSX website. Our baseline electron temperature seems to be $T_e = 20eV$. For the paper, we have adopted $n_e = 5 \times 10^{20} m^{-3}$ and we can fix the mean magnetic field at $\bar{B} = 0.1T$. This gives a $\beta = 0.4$ which is sensible. The volume inside the $0.4m$ diameter, $0.6m$ long flux conserver is $V = 0.075m^{-3}$ so we have the following baseline parameters.

$$N_{tot} = 3.77 \times 10^{19}$$

$$W_{mag} = 300J$$

$$W_{th,e} = 120J$$

If all the electrons eventually get heated to $T_e = 35eV$, then the energy stored in the electrons goes up to $W_{th,e} = 210J$. This represents an increase of $90J$ for the electron fluid (which comes at the expense of the magnetic field). Note that the way this works out,$W_{th,e} = W_{mag}$ so $\beta = 1$ at the end of the heating interval ($t = 70\ \mu s$).

Let's consider the heat source localized to the middle 10% of the volume (say $0.06m$ wide). What I have in mind is a small fraction of the density there (say 10% so $n_e = 5 \times 10^{19} m^{-3}$) is energized to 100 eV. To maintain particle balance, there's a little notch cut in the background density (see figure). This means there's a little over-pressure in the layer but that's ok since it could be supported by magnetic pressure on the outside.

I like the idea of a source of 100 eV electrons for a few reasons. (1) Our reconnection voltage is $EMF = 3mWb/30\mu s = 100V$ and its on for about $30\mu s$. (2) We see photons with the SXR array consistent with 100 eV. (3) 100 eV electrons have $v_{th} = 4m/\mu s, \nu_e = 20MHz, \lambda_{MFP} = 0.2m$ in the background plasma so they leave the line of sight quickly without a collision in the layer but subsequently and rapidly collide with the background plasma (20 times each μs). Another way to think about it is via the parallel thermal diffusivity (which is huge): $\chi_e \cong \lambda^2 \nu \cong 10^6\ m^2/s$ so T_e is likely uniform everywhere (local hot spots would be smoothed out in less than a μs.

The mean free path scales like the energy (or temperature) **squared** because $\lambda_{MFP} = v\tau_{coll} \sim E^{1/2} E^{3/2}$. If we drop the energy down to 30 eV or so, then $\lambda_{MFP} = 0.02m$ (for electrons... ions have a mean free path a factor of $\sqrt{2}$ larger). This means that the background (or even the warmed) electrons collide more often and have a high probability of collision within the reconnection zone.

The 100 eV electrons in the reconnection layer at $n_e = 5 \times 10^{19} m^{-3}$ have the following properties:

$$N_{tot} = 3.77 \times 10^{17}$$

$$W_{th,e} = 6J$$

Reconnection provides about $90J$ of energy (in the form of 100 eV electrons) in about 30 μs which corresponds to about 3 MW. The 100 eV electrons re-distribute their energy uniformly around the machine in less than a microsecond. The $6J$ of energy in the layer produced at a rate of 3 MW corresponds to a re-supply time of 3 μs. This means we have to re-energize electrons in the layer at that rate to keep a constant 3 MW. The transit time for a 30 eV electron is $\tau = 0.6m/2.2m/\mu s = 0.3\ \mu s$ so what we need is for 1 in 10 passing electrons to get energized to 100 eV.

This model is pretty simplistic but it hangs together and gives a simple result:

$$nk\frac{dT}{dt} = 3\ MW$$

So, we posit a 3 MW source of 100 eV electrons, localized at the midplane, on for 30 μs. 3 MW at 100 V corresponds to 30 kA which is also consistent with a reconnection layer of $B = 0.1T$ in a $0.2m$ radius cylinder. This heating source will also couple to the ions which could explain our slow increase in ion temperature too.

Proposed new paragraph (summary section):

A simple model explains our electron heating results. In figure 8 we see a nearly linear increase in electron temperature from 20 to 35 eV in 30 μs. Since electron thermal conduction is so high, we postulate that entire SSX electron inventory is heated. At a density of 5×10^{14} cm^{-3} and a plasma volume of 0.075 m^3 this heating corresponds to an energy increase of 90 J and a power of 3 MW. Neither an impulsive heat source nor a constant temperature source explain the linear increase so we posit a constant 3 MW power source on for the duration of the reconnection process. Earlier measurements of the reconnection rate [**?**] suggest an EMF of $\xi_{recon} = 3mWb/30\mu s = 100V$. We also have evidence of 100 eV electrons from the SXR array. A localized 3 MW source of 100 eV electrons has only about 10% of the background density explaining why we don't see electron heating immediately. In addition, 100 eV electrons have a mean free path of $\lambda_{MFP} \cong 0.2m$ and a collision frequency of $\nu_e = 20$ MHz so they rapidly leave the line of sight but quickly redistribute their energy among the bulk electrons. This model predicts a linear ramp of electron temperature from 20 to 35 eV in 30 μs, just what we observe. We are planning a more sophisticated Monte Carlo simulation of this process and more direct measurements of accelerated electrons.

Better model:

I think a more sophisticated model would have the probabilities built in (a Monte Carlo simulation). It could still be 1D with a 20 eV background Maxwellian distribution of electrons bouncing back and forth. In the middle would be a zone such that on average 1 in 10 electrons were promoted to 100 eV. In the bulk, the electrons would go a mean free path (on average) and re-distribute their energy. One could adjust the heating fraction (10%), energization energy (100 V for now), spatial distribution of heating zone (middle 10% for now), and maybe add a loss mechanism (none at the moment). I think the result would be basically the same as we have here. There are exact formulas we could use for collision rates of energetic electrons slowing down and exchanging energy with Maxwellian distributions of colder electrons and ions (see the Plasma Formulary for a start). We could even include the collision rates for excitation of the 1% carbon ions as well as coupling between the electrons and ions.

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