Magnetic Lifetime of Taylor State Plasmas

Our goal for Summer 2018 is to maximize the lifetime of our Taylor state configuration. Since the field structure (and therefore the current paths) are twisted, calculating the lifetime isn't trivial, but it all starts with L/R (inductance over resistance). First step is a simple estimate of lifetime of an axisymmetric spheromak (or FRC) to predict the scaling.

Scaling; Consider an axisymmetric spheromak torus sitting in a cylindrical can (flux conserver, see figure). The major and minor radii are both r, and the height of the cylinder is $2r$. The model we're adopting is that the magnetic lifetime is the inductive decay time: $\tau = L/R$. The inductance of a fat loop of current is to a good approximation $L = \mu_0 r$. The resistance is $R = 2\pi r\rho/\pi r^2$. So,

$$\frac{L}{R} = \frac{\mu_0 r \pi r^2}{2\pi r\rho} = \frac{\mu_0 r^2}{2\rho}$$

This shows the essential scaling. The lifetime scales like r^2 and inversely with resistivity. Since resistivity scales like $T^{-3/2}$, our goal is to increase temperature.

A good baseline Spitzer resistivity for SSX (see scalings notes) is $\rho = 1.6 \times 10^{-5}\ \Omega m$ (for $T_e = 10\ eV$), and our flux conserver radius is $r = 0.08\ m$. Plugging in numbers gives $\tau = 250\ \mu s$.

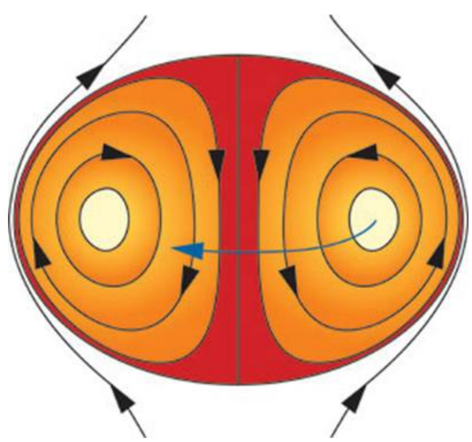

Figure 1: Simple axisymmetric spheromak (or FRC). Major and minor radii are both r. The height of the cylinder is $2r$,

A better model: We know that the current in a relaxed Taylor state flows in a twisted path (see figure). There are a few theoretical relationships that we can use. First the Taylor state fields obey the force-free eigenvalue condition: $\nabla \times B = \lambda B$. So we have:

$$\lambda = \frac{\nabla \times B}{B} = \frac{\mu_0 J}{B} = \frac{\mu_0 I}{\Phi}$$

where we have used Ampere's law, and integrated top and bottom over an area. The definition of inductance is: $\Phi = LI$ where Φ is the flux. So we have:

$$\lambda = \frac{\mu_0 I}{LI} = \frac{\mu_0}{L}$$

and a simple formula for inductance: $L = \mu_0/\lambda$. This is satisfying since we know that Taylor states seek the minimum λ, so that means they seek to maximize their inductance. From the Gray PRL, the infinite Taylor ground state has $\lambda r = 3.11$ (or $\lambda = 40\ m^{-1}$, for $r = 0.08\ m$).

The resistance is harder. Crudely, the resistance is the resistivity times some length, divided by some area. So R has the units of ρ/ℓ or $\lambda\rho$. This suggests a compact formula for the lifetime:

$$\tau = \frac{L}{R} = \frac{\mu_0}{\lambda^2 \rho}$$

Plugging in numbers gives $\tau = 50\ \mu s$, which is pretty close to what we measure. The problem with this formula is that is doesn't take into account the length, which should surely matter. The resistance should be something like $R = \rho(2\ell)/\pi r^2$, I think.

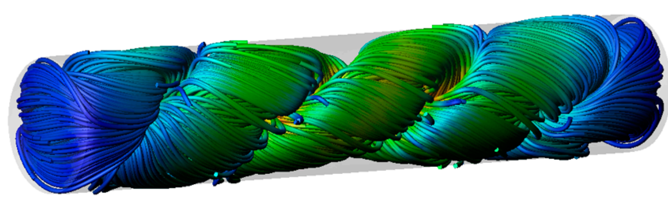

Figure 2: SSX Taylor state. Current flows in a twisted path, increasing the inductance and therefore lifetime for a given size.