SSX interferometer notes
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Preliminaries: The fundamental idea of plasma interferometry is that electromagnetic radiation (light) that passes through plasma has a wavelength different than it has in air or vacuum. The wavelength is actually longer in plasma. The index of refraction in plasma can be written:

\[ N_{\text{plasma}} = 1 - \frac{\omega_p^2}{2\omega_{\text{HeNe}}^2} = 1 - \frac{n}{2n_c} \]

where \( \omega_p^2 = n_e e^2/\varepsilon_0 m_e \) is the plasma frequency, and \( \omega_{\text{HeNe}} \) is the frequency of the radiation. We use a HeNe laser, so the wavelength is \( \lambda = c/f = 632.8 \times 10^{-9} \text{ m} \). It is sometimes helpful to write the laser frequency in terms of a “critical density” \( (n_c) \) i.e. a density such that the plasma frequency at \( n_c \) equals the laser frequency \( (n_c = \varepsilon_0 m_e \omega_{\text{HeNe}}^2/e^2) \). This formula works as long as the laser frequency is large compared to the plasma frequency. For us, the ratio \( \omega_{\text{HeNe}}/\omega_p \geq 100 \), so we’re good. Note also that \( \omega_{pe}/\omega_{ce} \geq 100 \) also so there is a large separation between electron magnetic fluctuations, electron electric fluctuations, and laser light fluctuations.

![Mach-Zehnder Interferometer Diagram](image)

Figure 1: Typical Mach-Zehnder interferometer set up. The scene and reference paths are identical except for the path \( L \) through the plasma. For SSX, \( L = 0.016 \text{ m} \).

Details: Since \( \lambda_{\text{HeNe}} = 0.6 \mu \text{m} \) and our path length is comparable to \( 0.6 \text{ m} \), we can think of there being about \( 10^6 \) wavelengths across the path.
With SSX plasma, and this interferometer, we change the path length by about 1 wavelength (ie one fewer wavelength in the plasma than in the vacuum). The path difference can be written:

$$\Delta s = \int_{z_1}^{z_2} N_{air} \, dz - \int_{z_1}^{z_2} N_{plasma} \, dz$$

Using the first formula and noting that \(N_{air} \approx 1\), and noting that the ratio of \(\phi\) compared to \(2\pi\) is the same as the ratio of \(\Delta s\) to the wavelength \(\lambda\), we get:

$$\phi = \frac{\pi}{\lambda n_e} \int_{z_1}^{z_2} n(z) \, dz = \frac{\lambda e^2}{4\pi c^2 \epsilon_0 m_e} \int_{z_1}^{z_2} n(z) \, dz$$

If you plug in the HeNe wavelength, electron charge and mass, \(\epsilon_0\), and speed of light (all MKS of course), you get for our typical peak density of \(n = 10^{22} \, m^{-3}\) and a path length of \(L = 0.16 \, m\):

$$\phi_{22} = 2.85 \, \text{radians}$$

Since a phase shift of \(\pi/2 = 1.57\) corresponds to one excursion from dark to bright (ie the full envelope), 2.85 radians corresponds to about twice the envelope voltage (ie about 1 volt for us, if the alignment is good). We typically measure \(n = 10^{21} \, m^{-3}\), so less than one fringe excursion.

References:

Hutchinson, Principles of Plasma Diagnostics