The number of protons in a Larmor sphere

As we think about physics below the proton gyro-scale, it's useful to think about how many particles are there. Note that the solar wind plasma has a Larmor radius of about 100 km, while the inter-particle spacing is about 1 cm. So, there are (on average) about 10^7 protons across a typical proton orbit in the solar wind. In SSX, that number is a little less.

I think what matters is the number of particles in a Larmor sphere (a sphere with radius ρ_i). I've never heard anyone use this phrase before. This should be compared to the number of particles in a Debye sphere (a sphere with radius λ_D) since plasma physics really doesn't apply inside λ_D. So we have:

$$N_{Lar} = \frac{4\pi}{3}\rho_i^3 n = \frac{4\pi}{3}\left(\frac{Mv_i}{qB}\right)^3 n$$

$$N_D = \frac{4\pi}{3}\lambda_D^3 n = \frac{4\pi}{3}\left(\frac{\epsilon_0 kT}{nq^2}\right)^{3/2} n$$

What's interesting is their ratio:

$$\frac{N_{Lar}}{N_D} = \frac{M^3 v_i^3}{q^3 B^3}\frac{n^{3/2}q^3}{\epsilon_0^{3/2} kT^{3/2}} = \frac{M^3 kT^{3/2}}{B^3 M^{3/2}}\frac{n^{3/2}}{\epsilon_0^{3/2} kT^{3/2}}$$

We get finally:

$$\frac{N_{Lar}}{N_D} = \frac{M^{3/2} n^{3/2}}{\epsilon_0^{3/2} B^3} = \left(\frac{c}{v_{Alf}}\right)^3$$

because $v_{Alf} = B/\sqrt{\mu_0 M n}$. This is an interesting result, and a very big number is most cases. For SSX, $c/v_{Alf} \approx 2000$, so $N_{Lar}/N_D \approx 8 \times 10^9$.