The contribution of cyclotron heating induced spatial modulation of electron magnetization to tokamak current drive

M. A. Hayes and M. R. Brown
Department of Physics, Dartmouth College, Hanover, New Hampshire 03755

(Received 14 May 1985; accepted 3 October 1985)

Application to a tokamak of electron-cyclotron resonant heating (ECRH) injected at a given poloidal angle will result in a small but important variation of average electron magnetic moment with poloidal angle. This, coupled with the dependence of magnetic field strength on poloidal angle, will result in the transmission of a net toroidal force to the electrons, the direction and magnitude of which depend on the poloidal angle at which the ECRH is injected and the direction of velocity of the electrons by which it is absorbed. The geometries for which this mechanism adds to the Fisch–Boozer mechanism [Phys. Rev. Lett. **45**, 720 (1980)] are described, as are those for which this mechanism is in opposition. Taking this mechanism into account, it is demonstrated that electrons at large minor radii can be driven with significantly greater efficiency if resonance is on the inboard side of the tokamak than if it is on the outboard side, and the ratio of the two efficiencies is calculated for the case of Coulomb collisionality in the limit of large Larmor radius. The case of turbulence-dominated collisionality is considered, and a general prediction of the ratio of the efficiency of this drive mechanism to that of the Fisch–Boozer mechanism is presented, in which the ratio of the efficiencies is given in terms of the dependence of the slowing-down time on velocity. The mechanism described herein provides a significant correction to predictions of current drive efficiency for infinite Larmor radius Coulomb collisionality, and is likely to provide a more significant correction yet for more realistic models of collisionality.

I. INTRODUCTION

There is a great deal of interest in the prospect of driving tokamak currents by noninductive means.[1,2,3] In 1980, Fisch and Boozer proposed injecting electron-cyclotron resonant heating (ECRH) into a tokamak in order to drive a current by asymmetrizing the resistivity of the plasma as a result of preferential heating of the electrons going in one direction.[2] In 1981, three different methods of driving current by using ECRH in conjunction with the confining magnetic field to directly impose a net toroidal force on the electrons in a tokamak were independently suggested, two by Hayes and De Groot,[4] and one by Parks and Marcus.[5]

The method suggested by Parks and Marcus requires a localized distortion of the toridal magnetic field (in the form of a single toroidal "bump" in the magnetic field, where the field would be required to be about 1.15 times the strength of the field at other toroidal locations), as well as a *toroidally* asymmetric application of ECRH. Parks and Marcus calculated a current drive efficiency for this unusual geometry (in the limit of infinite major radius), but this calculation is not applicable to normal tokamaks.

The more promising of the two methods suggested by Hayes and De Groot involves no distortion of tokamak magnetic fields, but instead depends on the variation in magnetic field strength along a field line, which results from the interaction of finite rotational transform with finite major radius. This variation in magnetic field strength, coupled with a *poloidally* asymmetric application of ECRH, can be used to drive a net toroidal current (the other of the methods suggested by Hayes and De Groot is similar in principle to the method suggested by Parks and Marcus, but makes use of the small distortions present in even the most carefully designed tokamak, rather than intentionally distorting the field). The present paper provides a significant elaboration on the finite rotational transform current drive suggestion of Hayes and De Groot, and includes a calculation of the current drive efficiency of this method; this calculation is directly applicable to real tokamaks in the standard configuration.

We will show that for ECRH absorbed at finite minor radius, parallel forces resulting from spatial modulation of the average electron magnetic moment will be significant, and will result in a current drive of comparable magnitude (at large minor radii) to that resulting from the Fisch–Boozer mechanism. This additional drive mechanism will be shown to add to that of Fisch–Boozer for ECRH injection from the inboard side of the tokamak, but to subtract from it for ECRH injection from the outboard side.

We will demonstrate that the current driven by the mechanism under consideration here is about a sixth of that resulting from the resistivity asymmetrization of Fisch and Boozer, assuming Coulomb collisionality in the infinite Larmor radius limit (the collisionality model used in the derivation of the Fisch–Boozer efficiency). It is possible to gain an intuitive understanding of why these two mechanisms, which at first glance are so dissimilar, give such similar results. In any Doppler-shifted resonance current drive, a group of particles of velocity v, which is making some contribution to the total current, anyway, is given extra energy, and in return is expected to make a larger (or in some cases, smaller) contribution to the current. The current contributed by a particle of velocity v is proportional to v, but it is also proportional to τ, the slowing-down time, that is, it is proportional to the product $v\tau$, which is just the distance traveled by the charged particle before its velocity is rando-

 0031-9171/86/010247-05$01.90

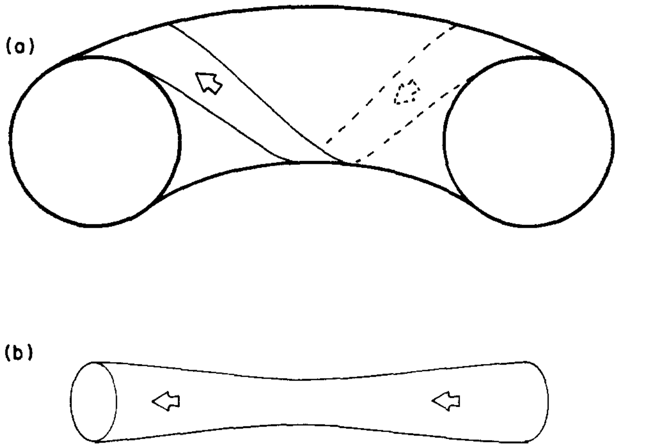

FIG. 1. Part (a) traces a flux tube around a section of a tokamak (the relative strength of the poloidal field is greatly exaggerated for clarity). Part (b) shows the same tube unwound from the tokamak.

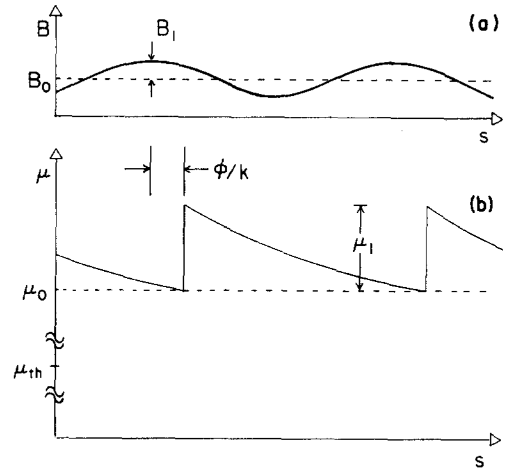

FIG. 2. Part (a) shows the variation of magnetic field strength with s, the displacement along the flux tube. Part (b) shows the variation of $\langle \mu \rangle$ with s for right-going electrons of velocity v (which Doppler shifts them into resonance with the ECRH). Note that resonance occurs a distance ϕ/k to the right of the position of maximum B.

mized. Thus I/P, the current per unit power, should vary as $d(v\tau)/dW$, where W is the kinetic energy of the particle, or

$$\frac{I}{P} \propto \left[\frac{d(v\tau)}{dW}\right] \propto \left(\frac{1}{v}\right)\left[\frac{d(v\tau)}{dv}\right]$$
$$= \left(\frac{1}{v}\right)\left[\tau + v\left(\frac{d\tau}{dv}\right)\right] = \left(\frac{\tau}{v}\right) + \left(\frac{d\tau}{dv}\right),$$

where we have assumed that the entire increase in kinetic energy is accounted for by an increase of the velocity component in the parallel direction. The first term (τ/v) is the increase in current that is obtained directly by increasing the speed of the charge carrier, while the second term is the increase in current that is obtained by the increase in τ that results from the reduction of collisionality at higher energy. Since for Coulomb collisionality in the infinite Larmor radius limit, $\tau \approx v^3$, $(d\tau/dv) = 3(\tau/v)$, so the effect of direct drive techniques (such as the magnetization modulation technique presented here) may be expected to be about one-third that of asymmetric resistivity (if all kinetic energy is parallel energy), provided that the system is characterized by either Coulomb collisionality in the infinite Larmor radius limit or some other set of dissipative processes for which $\tau \approx v^3$. In a turbulent plasma,[6] it is quite possible that wave–particle interactions would predominate over Coulomb collisions. In this case, the ratio of the efficiency of asymmetric resistivity current drive to the efficiency of magnetization modulation current drive would be (roughly) $2n$, where n is just that power of velocity proportional to the slowing-down time; the factor of 2 comes from the fact that, for reasonable choices of tokamak parameters, the change in average parallel energy is about half the added perpendicular energy. As will be shown below, the two effects are in opposition for resonance on the outboard side of the tokamak. In this geometry, it should be noted that should the effective n fall below $\frac{1}{2}$, e.g., because the current itself was driving up an unstable wave, the direction of current drive would *reverse*. We will show that this rather spectacular possibility can be avoided by applying the ECRH such that the resonance is on the inboard side of the tokamak.

II. THEORY

We will parametrize position along a magnetic field line by s. If the field line in question is at a finite minor radius, then, because of Ampere's law, the strength of the magnetic field will be a periodic function of s, as shown in Fig. 1. If $j = (2\pi/\text{rotational tranform})$, i.e., the number of times the field line goes around toroidally, for one poloidal circuit, then we may approximate the magnetic field strength as a function of s:

$$B(s) = B_0 + B_1 \cos(ks + \phi), \tag{1}$$

where B_0 and B_1 are constant, and $k = 2\pi/L$, where L is the total distance that must be traveled along a field line in order to return to the original poloidal angle. Thus, $L = 2\pi Rj$, so $k = 1/Rj$. Equation (1) just represents the variation of the strength of B, as the field line travels between small and large major radii. A plot of B vs s is presented in Fig. 2(a).

We must now consider how the average magnetic moment of a group of electrons of velocity v, which is in resonance with the ECRH, will vary with poloidal angle, and hence with s. If resonance with the ERCH occurs with a relatively small spread in poloidal angle, then the problem of the variation of $\langle \mu \rangle$ with s can be treated in a fairly straightforward fashion. If we call μ_{th} the average thermal magnetic moment of the background of unheated electrons, μ_0 the average magnetic moment of the resonant electrons just before they enter the resonance region, and μ_1 the average increase in magnetic moment of electrons of velocity v resulting from ECRH, then the plot of average magnetic moment $\langle \mu \rangle$ of electrons with velocity v will appear as shown in Fig. 2(b), where it has been assumed that the heated electrons have reached a steady state. That is, they have been heated to the point where their average loss of magnetic moment between resonance points just equals their average gain of energy within the resonance region. Now the difference between the average magnetic moment at a given point and the thermal value can be expressed as $\langle \mu(s) - \mu_{th} \rangle$. This difference should decrease exponentially with time, as Coulomb collisions and collective interactions allow exchange of energy and momentum between the resonant electrons and all other particles. Since we are looking at a group of electrons that are all moving with the same velocity v, we can translate this relaxation time into a relaxation length: $L_{re} = [v]$[relaxation time]. [Note that $v<0$ implies $L_{re}<0$. Inverting the sign of L_{re} corresponds to taking the mirror image of Fig. 2(b).] Thus, we obtain the differential equation $d\langle \mu(s) - \mu_{th}\rangle/ds = -\langle \mu(s) - \mu_{th}\rangle/L_{re}$, with the boundary conditions that $\langle \mu(0)\rangle = \mu_0 + \mu_1$ and $\langle \mu(L)\rangle = \mu_0$. The solution to this differential equation is

$$\langle \mu(s)\rangle = (\mu_0 + \mu_1 - \mu_{th})[\exp(-s/L_{re}] + \mu_{th}, \quad (2)$$

where Eq. (2) holds for $0<s<L$. For $s>L$, $\langle \mu(s)\rangle = \langle \mu(s-L)\rangle$ [see Fig. 1(a)]. Since we have assumed steady state, for all possible values of μ_1, μ_{th}, L, and L_{re}, μ_1 will be determined by the following equation:

$$\mu_0 - \mu_{th} = (\mu_0 + \mu_1 - \mu_{th})[\exp(-L/L_{re})]. \quad (3)$$

Now that we have forms for both $B(s)$ and $\langle \mu(s)\rangle$, we can calculate the average of the force on passing electrons $F(s) = -\langle \mu(s)\rangle(dB/ds)$. The average is taken over all passing electrons within a given flux tube. Thus,

$$F_{av} = -\left(\frac{1}{L}\right)\int_{s=0}^{L} [w(s)]\langle \mu(s)\rangle\left(\frac{dB}{ds}\right)ds, \quad (4)$$

where $w(s)$ is a factor that weights the force at s_0 by the number of passing electrons within the flux tube between s_0 and $s_0 + ds$. This weighting factor is inversely proportional to the parallel speed at s_0. We will calculate F_{av} under the approximation that $w(s) = 1$. This approximation is particularly good for very fast passing electrons (which will be shown to make the most important contribution to current drive efficiency), but it is reasonable even for slower electrons since most of the force is transmitted in regions where the strength of B changes most rapidly, and it is in just these regions that the weighting factor is closest to unity.

It is important to recall that F_{av} is the net average force on *passing* electrons; electrons that lie outside the loss cone oscillate in direction, and are unable to pick up net parallel momentum from F_{av}. For injection of ECRH from the inboard side of the tokamak, all electrons are passing electrons, since they have already made it to the maximum field strength possible along their flux tube. However, for injection of ECRH at other poloidal angles, the distinction between passing and trapped electrons is important, though it is reasonable to assume that few of the electrons with very high parallel velocity (which, as noted above, are the most effective current carriers) will be trapped.

Thus, the equation for the net average parallel force on electrons of velocity v within a specific flux tube may be written as

$$F_{av} = \left(\frac{kB_1}{L}\right)\int_{s=0}^{L}\left\{[\mu_0 + \mu_1 - \mu_{th}]\left[\exp\left(-\frac{s}{L_{re}}\right)\right] + \mu_{th}\right\} \times \sin(ks-\phi)ds. \quad (5)$$

No contribution to the integral is made by the constant part of μ, so defining a composite magnetic moment $\mu_c = \mu_0 + \mu_1 - \mu_{th}$, the integral simplifies to

$$F_{av}(kB_1\mu_c/L)[(\cos\phi)I_s - (\sin\phi)I_c], \quad (6)$$

where

$$I_c = \int_{s=0}^{L}\left[\exp\left(-\frac{s}{L_{re}}\right)\right]\cos(ks)ds = \{L_{re}/[1+(kL_{re})^2]\}[1-\exp(L/L_{re})]$$

and

$$I_s = \int_{s=0}^{L}\left[\exp\left(-\frac{s}{L_{re}}\right)\right]\sin(ks)ds = \{k(L_{re})^2/[1+(kL_{re})^2]\}[\exp(-L/L_{re})].$$

By combining Eq. (3) with the definition for μ_c, we obtain

$$(\mu_1/\mu)_c = [1-\exp(-L/L_{re})]. \quad (7)$$

Thus, defining the dimensionless parameter $a = kL_{re}$, the above forms reduce to $I_c(1/k)[a/(1+a^2)](\mu_1/\mu_c)$ and $I_s = (1/k)[a^2/(1+a^2)](\mu_1/\mu_c)$. Therefore, we can write the equation for average parallel force on electrons of a specific parallel velocity as

$$F_{av} = (B_1\mu_1/L)[a^2/(1+a^2)][(\cos\phi) - (1/a)(\sin\phi)]. \quad (8)$$

As we have defined the problem, a positive force increases the magnitude of the average electron speed, while a negative force decreases it. As noted above, inverting the sign of v means inverting the sign of L_{re} and hence of a; note that the $(1/a)(\sin\phi)$ term in Eq. (8) changes sign when v is inverted, while the $(\cos\phi)$ term does not.

III. ASYMPTOTIC FORMS FOR THE AVERAGE FORCE IN THE HIGH AND LOW COLLISIONALITY LIMITS

A. High collisionality, $a^2 \ll 1$

For a very collisional plasma, $a^2 \ll 1$. This case is not relevant to a fusion experiment, but is an interesting regime to consider because of the dependence of the $(1/a)(\sin\phi)$ term on the sign of the velocity, which was noted above. For $a^2 \ll 1$, Eq. (8) reduces to

$$F_{av} = (-B_1\mu_1/L)[a/(1+a^2)](\sin\phi). \quad (9)$$

Since this scalar force was defined such that a positive F_{av} tends to increase the magnitude of the parallel speed of the particles, while a negative F_{av} tends to decrease it, the fact that the force in Eq. (9) changes sign with v indicates that particles with velocity v_0 are sped up, while those with velocity $-v_0$ are slowed down by an equal amount. Thus Dopper-shift techniques are not necessary in order to produce a net charged particle drive in this regime. The fact that this particle is not Doppler-shift dependent is important since a plasma sufficiently collisional that $a^2 \ll 1$ would probably be unlikely to contain particles moving fast enough to make the Doppler shift significant.

B. Low collisionality, $a^2 \gg 1$

For a fusion experiment, the relaxation lengths can be expected to be much greater than the scale length over which the magnetic field changes, that is, $a^2 \gg 1$ (the mean free path for Coulomb collisions, for instance, would typically be many kilometers for a fusion experiment). In this limit, Eq. (8) reduces to

$$F_{av} = (B_1 \mu_1 / L)(\cos \phi). \quad (10)$$

Thus, in this limit, F_{av} is independent of a, and the Doppler shift must be relied upon to distinguish between electrons of positive and negative parallel velocity, since otherwise both would be either sped up or slowed down, and hence no net current driven. In fusion experiments, thermal velocities are sufficiently high that distinguishing between electrons traveling in opposite directions is not difficult.

The sign of F_{av} is determined by $\cos \phi$ (see Fig. 2); for $\phi = 0$, the parallel speed of the resonant electrons is increased, and for $\phi = \pi$ it is decreased. Injection of ECRH from the inboard side of a tokamak corresponds to $\phi = 0$, while injection from the outboard side corresponds to $\phi = \pi$. In light of this, the dependence of F_{av} on ϕ becomes intuitively clear. At $\phi = 0$, the electron is at the maximum of magnetic field along its field line; increasing the electron's magnetic moment here increases the parallel speed of the electron at all other points along its path, and hence increases the magnitude of the average parallel momentum of the electron. Similarly, at $\phi = \pi$, the electron is at the minimum of magnetic field along its field line; increasing the electron's magnetic moment here *decreases* the parallel speed of the electron at all other points along its path, thus decreasing the magnitude of the average momentum of the electron. Therefore, this mechanism increases the speed of the resonant electrons if resonance is on the inboard side, and decreases the speed of the resonant electrons if resonance is on the outboard side. Thus, for inboard-side resonance, this drive mechanism will *add* to the Fisch–Boozer current, while for outboard-side resonance, this mechanism will *subtract*.

IV. CURRENT DRIVE EFFICIENCY

Equation (10) provides us with a net average force (in the limit of large a^2, which is appropriate for a tokamak) per passing electron of parallel velocity v. In order to calculate the efficiency of this current drive mechanism, we must both relate F_{av} to current and calculate the rf power required by the mechanism. The power is easily calculated. As we have defined the problem, the net average increase in magnetic moment μ as a particle passes the ECRH is simply μ_1, thus the average energy input per particle per pass through resonance is $\langle \Delta W_\perp \rangle = \mu_1 B_{res}$, where B_{res} is the strength of the magnetic field in the resonance region. The power P_{rf} is just the average energy change per particle, times the particle flux, times the cross-sectional area A of the resonant flux tube, or

$$P_{rf} = A\,(nv)\mu_1 B_{res}, \quad (11)$$

where n is the density of particles that are resonant with the rf. Combining Eqs. (10) and (11) allows us to eliminate μ_1 from the equation, and gives

$$F_{av} = (B_1/B_{res})(P_{rf}/vnAL)(\cos \phi). \quad (12)$$

However, L is the length that must be traversed along a field line before returning to the original poloidal angle, so $L = 2\pi R \xi$, where R is the major radius of the tokamak and ξ is the number of toroidal circuits that must be made in order to complete one poloidal circuit.

To find the average drift velocity as a function of power, we must balance the force given by Eq. (12) against the net drag on the resonant electrons resulting from collisions both with ions and with other electrons. Following the work of Spitzer,[7] we find the drag force per resonant electron (resulting from collisions against both the ions and the nonresonant electrons) to be

$$F = -(v)\nu_0 m_e [2x^2 G(x/\sqrt{2}) + 1], \quad (13)$$

where v is the velocity of the resonant electrons, x is the ratio of v to the electron thermal velocity,

$$\nu_0 = 8\pi e^4 n_0 (\ln \Lambda)/(m_e^2 v^3) \quad (14)$$

[with n_0 the density of ions (or background electrons) against which those electrons collide and $\ln \Lambda$ the Coulomb logarithm], and

$$G(x) = [\phi(x) - x\phi'(x)]/2x^2, \quad (15)$$

where $\phi(x)$ is the error function of x.

From Eqs. (12) and (13), we find that the condition for no net parallel acceleration of the resonant electrons is

$$v = \frac{(B_1/B_{res})(P_{rf}/vnAL)(\cos \phi)}{\nu_0 m_e [2x^2 G(x/\sqrt{2}) + 1]}. \quad (16)$$

However, current density $j = nev$, and $I = j(\xi A)$, as described above, and A is the cross-sectional area of the resonant flux tube. As defined above, ξ will be the number of times that a flux tube passes a fixed toroidal angle before returning to its original poloidal angle. Therefore, we can express the current as

$$I = \frac{e(B_1/B_{res})(P_{rf}/2\pi R v)(\cos \phi)}{\nu_0 m_e [2x^2 G(x/\sqrt{2}) + 1]}, \quad (17)$$

or, with T_{10} the temperature in units of 10 keV, N_{20} the density in units of $10^{20}\ m^{-3}$, and the rest of the units standard mks, we obtain

$$\frac{I}{P_{rf}} = \frac{(2.03 \times 10^{-2})(x^2 T_{10}/N_{20} R)(B_1/B_{res})(\cos \phi)}{2x^2 G(x/\sqrt{2}) + 1}. \quad (18)$$

Resonance on the inboard side of the flux tube corresponds to $\phi = 0$ and $B_1/B_{res} = B_1/(B_0 - B_1)$, while resonance on the outboard side corresponds to $\phi = \pi$ and $B_1/B_{res} = B_1/(B_0 + B_1)$. Using values that correspond to a flux tube for which $B_1 = B_0/3$, and taking $x \gg 1$, we obtain

$$I/P_{rf} = (-3.4 \times 10^{-3}) x^2 T_{10}/(N_{20} R) \quad (19)$$

for outboard-side resonance, where the minus sign indicates that this effect tends to retard the motion of the resonant electrons (and consequently oppose the asymmetric resistivity effect), and

$$I/P_{rf} = (+1.7 \times 10^{-3}) x^2 T_{10}/(N_{20} R) \quad (20)$$

for inboard side resonance. The asymmetric resistivity effect as calculated by Karney and Fisch[3] is almost exactly negative six times the result of Eq. (19), as predicted by the simple analysis presented in the Introduction (assuming that half of the added energy, on average, appears as parallel energy), which was based on the dependence of the slowing-down time on the velocity.

V. CONCLUSION

The periodic modulation of electron magnetization that results from the injection of ECRH at a given poloidal angle has been shown to make a significant contribution to the current drive efficiency. Because this mechanism adds to that of Fisch and Boozer for inboard-side resonance, and subtracts from it for outboard-side resonance, we predict outboard-side resonance to be about 30% more efficient than inboard-side resonance for driving current at large minor radii, based on Coulomb collisionality in the limit of large Larmor radius. Other models of collisionality, e.g., models of turbulent collisionality, that give a different dependence between the slowing-down time τ and the speed v will give different ratios. A comparison of the efficiencies of inboard and outboard resonance drives might provide a measurement of plasma turbulence, as the difference of the two efficiencies over their sum is a measure of $(\tau/v)/(d\tau/dv)$, which in turn depends on the nature of the predominant randomization mechanism in operation.

The results presented in this paper also apply to the ions (note that the particle mass does not enter into the force equation), with the appropriate small changes to the Coulomb collisionality. While the ions are too massive to be practical for current drive (the directed kinetic energy of the current-carrying ions would be greater than their thermal energy, with the attendant increase in β), situations in which it would be desirable to change the poloidal momentum of a class of ions may arise at some point. In such an eventuality, the technique presented here for the electrons should also work for the ions.

ACKNOWLEDGMENTS

Valuable conversions with D. C. Montgomery and N. J. Fisch are gratefully acknowledged.

[1] N. J. Fisch, Phys. Rev. Lett. **41**, 873 (1978); S. Bernabei, C. Daughney, P. Efthimion, W. Hooke, J. Hosea, F. Jobes, A. Martin, E. Mazzucato, E. Meservey, R. Motley, J. Stevens, S. Von Goeler, and R. Wilson, Phys. Rev. Lett. **49**, 1255 (1982); F. Jobes, J. Stevens, R. Bell, S. Bernabei, A. Cavallo, T. K. Chu, S. Cohen, B. Denne, P. Efthimion, E. Hinnov, W. Hooke, J. Hosea, E. Mazzucato, R. McWilliams, R. Motley, S. Suckewer, G. Taylor, J. Timberlake, S. Von Goeler, and R. Wilson, Phys. Rev. Lett. **52**, 1005 (1984).

[2] N. J. Fisch and A. H. Boozer, Phys. Rev. Lett. **45**, 720 (1980).

[3] C. F. F. Karney and N. J. Fisch, Phys. Fluids **28**, 116 (1985).

[4] M. A. Hayes and J. S. DeGroot, Phys. Lett. A **86**, 161 (1981).

[5] P. B. Parks and F. B. Marcus, Nucl. Fusion **21**, 1207 (1981).

[6] T. M. O'Neil, Phys. Rev. Lett. **25**, 995 (1970).

[7] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Wiley–Interscience, New York, 1962), 2nd ed.; L. Spitzer, Jr. and R. Harm, Phys. Rev. **89**, 977 (1953).